EEE 391 Recitation 2

1)

Use the Fourier series analysis equation (3.39) to calculate the coefficients a_k for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \leq t < 1 \\ -1.5, & 1 \leq t < 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

Since $\omega_0 = \pi$, $T = 2\pi/\omega_0 = 2$. Therefore,

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}dt$$

Now,

$$a_0 = \frac{1}{2}\int_0^1 1.5dt - \frac{1}{2}\int_1^2 1.5dt = 0$$

and for $k \neq 0$

$$\begin{aligned} a_k &= \frac{1}{2}\int_0^1 1.5e^{-jk\pi t}dt - \frac{1}{2}\int_1^2 1.5e^{-jk\pi t}dt \\ &= \frac{3}{2k\pi j}[1 - e-jk\pi] \\ &= \frac{3}{k\pi}e^{-jk(\pi/2)}\sin(\frac{k\pi}{2}) \end{aligned}$$

$$= \frac{3}{4}\int_0^1 e^{-jk\pi t}dt - \frac{3}{4}\int_1^2 e^{-jk\pi t}dt = \frac{-3}{4}\left(\frac{e^{-jk\pi t}}{jk\pi}\right)\Big|_0^1 + \frac{3}{4}\left(\frac{e^{-jk\pi t}}{jk\pi}\right)\Big|_1^2$$

$$= \frac{-3}{4}\left[\frac{e^{-jk\pi} - 1}{jk\pi}\right] + \frac{3}{4}\left[\frac{e^{-j2k\pi} - e^{-jk\pi}}{jk\pi}\right] = \frac{3}{4}\left[\frac{e^{-j2k\pi} - 2e^{-jk\pi} + 1}{jk\pi}\right]$$

$$= \frac{3}{4}\left[\frac{(e^{-j2\pi})^k - 2e^{-j\pi k} + 1}{jk\pi}\right] = \frac{3}{4}\left[\frac{1 - 2e^{-j\pi k} + 1}{jk\pi}\right] = \frac{3}{2}\left[\frac{1 - e^{-j\pi k}}{jk\pi}\right]$$

Since $e^{j2\pi} = 1 \Rightarrow (e^{-j2\pi})^k = 1$

$$= \frac{3}{2jk\pi}e^{-jk\frac{\pi}{2}}\left(e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}\right) = \frac{3}{2k\pi}e^{-jk\frac{\pi}{2}}\underbrace{\frac{1}{j}\left(e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}\right)}_{\sin(\frac{k\pi}{2})}$$

$$= \frac{3}{k\pi}\cdot e^{-jk\frac{\pi}{2}}\cdot \sin\left(\frac{k\pi}{2}\right) \quad //$$